Precalculus Honors 2012-2013 Summer Packet

Hello students! I want to thank you for your interest in the precalculus honors course. The function of this course is defined below:

Let x be a measure of student success with the Alg2 curriculum and its prerequisites,

$$f(x) = \begin{cases} AP\ Calculus\ Student & x \geq Adequately\ prepared\ Alg2\ Master \\ \emptyset & x < Adequately\ prepared\ Alg2\ Master \end{cases}$$

That is to say that the domain of this course includes students that have built a strong foundation with their Alg2 experience and are prepared to move forward in mastering problem solving pertaining to trigonometry, polynomials with any real roots, conic sections, parametric equations, and the polar number system, to prepare to master the first year of calculus during the following year.

I've found that students often return to school in the fall with a layer of "rust" that needs to be scoured away in order to discern whether a student is compatible with an advanced math course at an honors pace. This packet's purpose is to help students show up on the first day without struggling to recover that which they have already mastered.

This packet should contain **no new material**, provided you've had a strong learning experience in Alg2. If you discover that your Alg2 experience was lacking, you're welcome to contact me at jwytiaz@alpinedistrict.org for assistance. If you are significantly confused by the material included in this packet, it might be time to reconsider your pursuit of the precalculus curriculum at an honors pace.

Mr. Wytiaz

OHS Math Dept

I suggest using a 3-hole punch and inserting this packet into the same 3-ring binder that you'll use for note-taking and keeping materials for this course.

How to use this packet

I feel that the best results for students will come from following my pacing guide below. Realize that this is just a recommended pacing; have some fun over the summer! Adjust the pacing around your family vacations, movie marathons, or need to relax; just make sure your plan includes finishing the material by August 20th.

You are looking to be a successful student AND have fun. Your Alg2 experience should have covered the difference between the conjunctions AND and OR. Perhaps this summer will provide an opportunity to learn to manage your time to maximize both success AND fun. You'll certainly need time management skills to do well in your upper level academic course work.

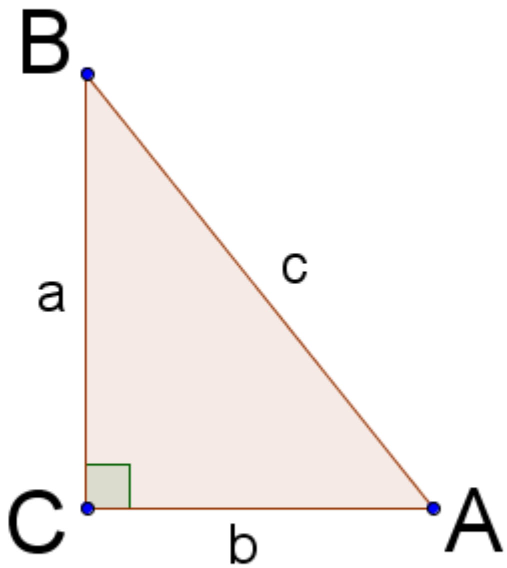
Week of summer "vacation"	**Suggested Review Material**
Week of June 4th	
Week of June 11th	Pythagorean Theorem
Week of June 18th	Trigonometric Functions
Week of June 25th	
Week of July 2nd	Factoring Quadratic Expressions
Week of July 9th	Factoring Special Cases
Week of July 16th	Solve by Factoring
Week of July 23rd	
Week of July 30th	Simplifying Rational Expressions
Week of August 6th	Rational w/ Holes and Vertical Asymptotes
Week of August 13th	

I suggest that plowing an entire worksheet in one sitting is not as impactful as sitting down for 20 minutes and doing 4 or 5 problems each day of the week. Having multiple exposures to a topic typically correlates well with assessments of how well the material "adheres" to the mind. Enjoy the light, leisurely pace; we won't move quickly until the school year begins.

The following page contains a few helpful pointers for students that are struggling with recalling their lessons from Alg2. Again, you are most welcome to contact me if you need a supplemental nudge in the right direction.

The packet will be graded and our first test will include material from this packet. A keen student would seize this opportunity to pad their grade with free points. Consider this your first opportunity to impress.

Pythagorean Theorem



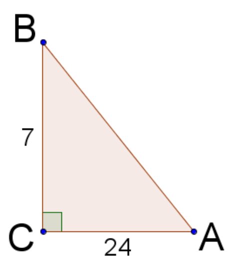
Named for the Greek mathematician, Pythagoras, this theorem relates the sides of a right triangle with the formula:

$$a^2 + b^2 = c^2$$

where a, b are the lengths of the legs of the right triangle and c is the length of the hypotenuse of the right triangle.

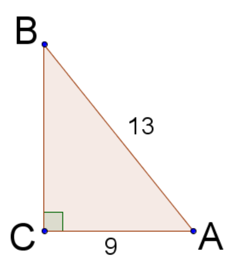
Points are represented by upper case letters. The side of the triangle opposite the angle at a given point is represented with the lower case version of the same letter (e.g., point B is directly across from the side with a length of b).

Example 1: Find the missing side length of the triangle:



$a^2 + b^2 = c^2$	Write the Pythagorean Theorem.
$7^2 + 24^2 = c^2$	Use information from the figure.
$49 + 576 = c^2$	Evaluate the exponentials.
$625 = c^2$	Addition.
$\pm 25 = c$	Square root each side.
$25 = c$	We prefer to consider distances as positive values, so we eliminate the negative solution.

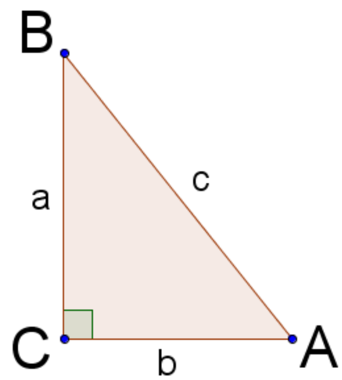
Example 2: Find the missing side length of the triangle:



$a^2 + b^2 = c^2$	Write the Pythagorean Theorem.
$a^2 + 9^2 = 13^2$	Use information from the figure.
$a^2 + 81 = 169$	Evaluate the exponentials.
$a^2 = 88$	Subtraction.
$a = \pm\sqrt{88}$	Square root each side.
$a = \pm 2\sqrt{22}$	Simplify the root.
$a = 2\sqrt{22}$	We prefer to consider distances as positive values, so we eliminate the negative solution.

Pythagorean Theorem

Find the missing side lengths of the right triangles using the Pythagorean Theorem given ΔABC:



1) $a = 5, b = 6$; Find c.

$c = \sqrt{61}$

2) $a = 6, b = 8$; Find c.

$c = 10$

3) $a = 5, b = 12$; Find c.

$c = 13$

4) $a = 7, c = 25$; Find b.

$b = 24$

5) $a = 3, c = 12$; Find b.

$b = 3\sqrt{15}$

6) $a = 30, c = 34$; Find b.

$b = 16$

7) $b = 5, c = 5\sqrt{2}$; Find a.

$a = 5$

8) $b = 8, c = 16$; Find a.

$a = 8\sqrt{3}$

9) $b = 2.2, c = 8.7$; Find a.

$a \approx 8.4$

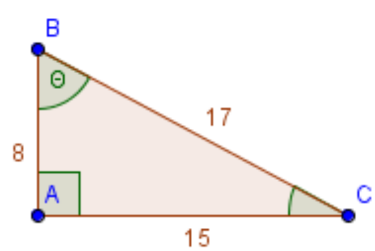
Trigonometric Functions

- A triangle with angle measurements of $45°, 45°$, and $90°$ has two legs of the same length and a hypotenuse with a length that is $\sqrt{2}$ times the length of either leg.
- A triangle with angle measurements of $30°, 60°$, and $90°$ has two legs of different lengths. The longer of the legs has a leg that is $\sqrt{3}$ times the length of the shorter leg. The length of the hypotenuse is twice as long as the shorter leg.

- A trigonometric ratio is a fraction that compares two sides of a triangle from the point of view of an angle. Each of the six trigonometric ratios has a unique comparison.
    - Sine is the ratio of $\frac{Opposite\ Leg}{Hypotenuse}$ and is written $\sin\theta$, where θ is an angle.
    - Cosine is the ratio of $\frac{Adjacent\ Leg}{Hypotenuse}$ and is written $\cos\theta$, where θ is an angle.
    - Tangent is the ratio of $\frac{Opposite\ Leg}{Adjacent\ Leg}$ and is written $\tan\theta$, where θ is an angle.
    - Cosecant is the ratio of $\frac{Hypotenuse}{Opposite\ Leg}$ and is written $\csc\theta$, where θ is an angle.
    - Secant is the ratio of $\frac{Hypotenuse}{Adjacent\ Leg}$ and is written $\sec\theta$, where θ is an angle.
    - Cotangent is the ratio of $\frac{Adjacent\ Leg}{Opposite\ Leg}$ and is written $\cot\theta$, where θ is an angle.

Example:

Find the value of each trigonometric ratio.



From θ's point of view:
- the opposite side's length is 15 units
- the adjacent side's length is 8 units
- the hypotenuse's length is 17 units

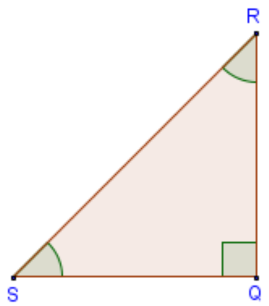
Therefore,

$\sin\theta = \frac{15}{17}$, $\cos\theta = \frac{8}{17}$, $\tan\theta = \frac{15}{8}$

$\csc\theta = \frac{17}{15}$, $\sec\theta = \frac{17}{8}$, $\cot\theta = \frac{8}{15}$

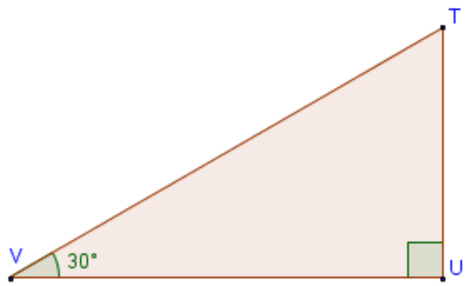
Trigonometric Functions

Find the value of each segment or angle.



$QR = 4$

Find QS, RS



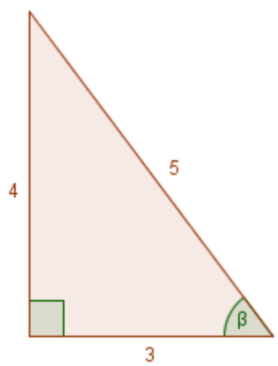
$UT = 5$

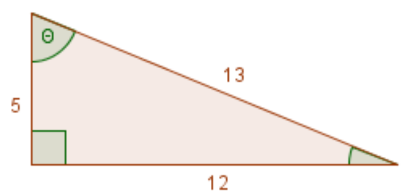
Find UV, TV

$QS = 4, RS = 4\sqrt{2}$

$UV = 5\sqrt{3}, TV = 10$

Find the value of each trigonometric ratio.





$\sin\beta = \frac{4}{5}$ $\csc\beta = \frac{5}{4}$ $\sin\theta = \frac{12}{13}$ $\csc\theta = \frac{13}{12}$

$\cos\beta = \frac{3}{5}$ $\sec\beta = \frac{5}{3}$ $\cos\theta = \frac{5}{13}$ $\sec\theta = \frac{13}{5}$

$\tan\beta = \frac{4}{3}$ $\cot\beta = \frac{3}{4}$ $\tan\theta = \frac{12}{5}$ $\cot\theta = \frac{5}{12}$

Factoring Quadratic Expressions

- When factoring an expression of the form $ax^2 + bx + c$:
  - if $a = 1$, then determine two factors (let's call them f_1 and f_2) of c that sum to b and write the factorization as: $(x + f_1)(x + f_2)$.
  - if $a \neq 1$, then determine two factors (let's call them f_1 and f_2) of the product ac that sum to b and rewrite the expression as: $ax^2 + f_1x + f_2x + c$, then factor by grouping.

Example 1:

Factor $x^2 + 5x - 24$

Since $a = 1$, I must find the factor pair of -24 that adds up to $+5$.

The only factor pair of -24 that sums to 5 is $f_1 = 8, f_2 = -3$.

So the factorization is $(x + 8)(x - 3)$

Example 2:

Factor $3x^2 + 11x + 10$

Since $a \neq 1$, I must find the factor pair of $3 \cdot 10$ that adds up to $+11$.

The only factor pair of 30 that sums to 11 is $f_1 = 5, f_2 = 6$.

So we rewrite the original expression as $3x^2 + 5x + 6x + 10$ and factor by grouping as follows:

$(3x^2 + 5x) + (6x + 10)$	Associate the first two terms and last two terms
$x(3x + 5) + 2(3x + 5)$	Factor the GCF from each association
$(x + 2)(3x + 5)$	Associate the "coefficients" of like terms

Your Alg2 practice should have enabled you to casually glance at a quadratic expression and write the factorization down quickly. The above algorithms should only be executed if a student is struggling with a particular problem.

Factoring Quadratic Expressions

Factor completely.

1)	$x^2 + 6x + 8$ $(x+2)(x+4)$	2)	$x^2 - 7x - 18$ $(x+2)(x-9)$	3)	$x^2 - 5x - 14$ $(x+2)(x-7)$
4)	$x^2 - 16x + 63$ $(x-9)(x-7)$	5)	$x^2 - 15x + 56$ $(x-7)(x-8)$	6)	$x^2 - 9x + 8$ $(x-8)(x-1)$
7)	$x^2 - 15x + 36$ $(x-12)(x-3)$	8)	$x^2 + 22x + 40$ $(x+2)(x+20)$	9)	$x^2 - x - 90$ $(x-10)(x+9)$
10)	$2x^2 + 9x + 7$ $(2x+7)(x+1)$	11)	$3x^2 - 4x - 4$ $(3x+2)(x-2)$	12)	$5x^2 + 12x + 4$ $(5x+2)(x+2)$
13)	$4x^2 + 15x - 4$ $(4x-1)(x+4)$	14)	$10x^2 + 23x - 5$ $(5x-1)(2x+5)$	15)	$6x^2 - 7x - 20$ $(2x-5)(3x+4)$
16)	$30x^2 + 7x - 1$ $(3x+1)(10x-1)$	17)	$12x^2 - 8x + 1$ $(6x-1)(2x-1)$	18)	$4x^2 + 12x + 9$ $(2x+3)^2$

Factoring Special Cases

- When factoring an expression of the form $a^2 - b^2$:
  - Write the factorization as: $(a+b)(a-b)$.

- When factoring an expression of the form $a^2 + 2ab + b^2$:
  - write the factorization as: $(a+b)(a+b)$ or $(a+b)^2$

- When factoring an expression of the form $a^2 - 2ab + b^2$:
  - write the factorization as: $(a-b)(a-b)$ or $(a-b)^2$

Example 1:

Factor $x^2 - 16$

As x^2 and 16 are perfect squares and there is a – sign between them,

the factorization is $(x+4)(x-4)$

Example 2:

Factor $4x^2 + 12x + 9$

As $4x^2$ and 9 are perfect squares and the middle term is equal to twice the product of their roots $(2 \cdot 2x \cdot 3 = 12x)$,

the factorization is $(2x+3)^2$

Example 3:

Factor $x^2 - 10x + 25$

As x^2 and 25 are perfect squares and the middle term is equal to the opposite of twice the product of their roots $(2 \cdot x \cdot 5 = -10x)$,

the factorization is $(x-5)^2$

For best results, remember to factor out the GCF of the expression before you do anything else!

Factoring Special Cases

Factor completely.

1) $x^2 - 16$

$(x+4)(x-4)$

2) $x^2 - 49$

$(x+7)(x-7)$

3) $x^2 - 121$

$(x+11)(x-11)$

4) $x^2 - 8x + 16$

$(x-4)^2$

5) $x^2 - 10x + 25$

$(x-5)^2$

6) $x^2 + 6x + 9$

$(x+3)^2$

7) $x^2 - 100$

$(x+10)(x-10)$

8) $x^2 - 1$

$(x+1)(x-1)$

9) $x^2 - 400$

$(x+20)(x-20)$

10) $x^2 + 12x + 36$

$(x+6)^2$

11) $x^2 - 7x + \frac{49}{4}$

$\left(x - \frac{7}{2}\right)^2$

12) $x^2 + 20x + 100$

$(x+10)^2$

13) $4x^2 - 64$

$4(x+4)(x-4)$

14) $10x^2 - 90$

$10(x+3)(x-3)$

15) $x^4 - 16$

$(x^2+4)(x+2)(x-2)$

16) $4x^2 - 4x + 1$

$(2x-1)^2$

17) $100x^2 - 60x + 9$

$(10x-3)^2$

18) $4x^2 + 12x + 9$

$(2x+3)^2$

Solve by Factoring

- When solving a quadratic equation, bring all terms to one side of the equation so that the other side is zero. Then, factor the quadratic. As the product of multiple factors equaling zero is only possible by a factor being zero, set each factor equal to zero and solve these simpler equations to find the solutions to the original equation.

Example 1:

Solve for x: $x^2 + 4x = -3$

$x^2 + 4x + 3 = 0$		Bring all terms to the left side.
$(x + 3)(x + 1) = 0$		Factor the quadratic.
$x + 3 = 0$	$x + 1 = 0$	Set each factor equal to zero.
$x = -3$	$x = -1$	Solve each equation.
$x = \{-3, -1\}$		Write the solutions to each equation in one solution set.

Example 2:

Solve for x: $x^3 + 4x^2 + x = 4x^2 + 26x$

$x^3 - 25x = 0$			Bring all terms to the left side.
$x(x^2 - 25) = 0$			Factor the GCF from the group.
$x(x - 5)(x + 5) = 0$			Factor the quadratic.
$x = 0$	$x - 5 = 0$	$x + 5 = 0$	Set each factor (including the GCF) equal to zero.
$x = 0$	$x = 5$	$x = -5$	Solve each equation.
$x = \{-5, 0, 5\}$			Write the solutions to each equation in one solution set.

Solve by Factoring

Solve by factoring.

1) $x^2 - 8x + 7 = 0$

$x = \{1, 7\}$

2) $x^2 + 6x + 5 = 0$

$x = \{-5, -1\}$

3) $x^2 - 12x + 20 = 0$

$x = \{2, 10\}$

4) $6x^2 + 13x + 6 = 0$

$x = \{-\frac{3}{2}, -\frac{2}{3}\}$

5) $x^2 - 36 = 0$

$x = \{-6, 6\}$

6) $x^2 - 10x + 25 = 0$

$x = 5$

7) $2x^2 + 17x = -21$

$x = \{-7, -\frac{3}{2}\}$

8) $7x^3 - 28x = 45x^2$

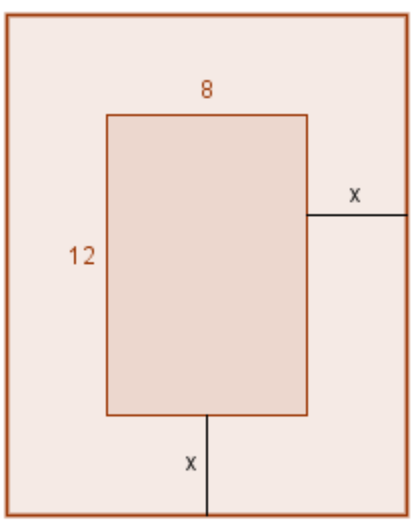
$x = \{-\frac{4}{7}, 0, 7\}$

9) $x^4 - 81 = 0$

$x = \{-3, 3, -3i, 3i\}$

10) Mrs. Shirk teaches in a rectangular classroom where the width of the classroom is 4 feet longer than the length of the classroom. The area of the classroom is 672 square feet. What are the dimensions of the classroom?

$l = 24, w = 28$



11) Mr. Elison has a rectangular garden that is 8 feet wide by 12 feet long. He decides to add a walkway around the garden that has the same width to form a larger rectangle. The area of the larger rectangle will be 224 square feet. What is the width of the walkway?

$x = -5 + \sqrt{57} \approx 2.55$

Solve by Completing the Square

- When a quadratic equation cannot be factored using integer or even rational values, completing the square is a method for solving for x. The method involves forcing $x^2 + bx + c$ to become $\left(x + \frac{b}{2}\right)^2$ so that square-rooting both sides of the equation sets the solver free from exponents. The technique requires no creativity; determination and accuracy will suffice.

Example 1:

Solve for x: $x^2 + 3x = -1$

$x^2 + 3x \qquad = -7$	Leave space for a constant term on the side with the quadratic.
$x^2 + 3x + \frac{9}{4} = -7 + \frac{9}{4}$	Divide b by 2 (in this example, $b = 3$ because 3 is the coefficient of x) and then square that quotient. We typically call this result c (in this example, $c = \frac{9}{4}$ because $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$). Add c to both sides, thus completing the square.
$\left(x + \frac{3}{2}\right)^2 = \frac{-19}{4}$	Rewrite the left side as a perfect square, $\left(x + \frac{b}{2}\right)^2$. Combine like terms on the right side, which typically requires finding a common denominator.
$x + \frac{3}{2} = \pm\frac{i\sqrt{19}}{2}$	Square-root both sides of the equation. This will always result in a $\pm$ symbol on the right hand side.
$x = -\frac{3}{2} \pm \frac{i\sqrt{19}}{2}$	Subtract the term that was preventing x from being alone. If the terms on the right side of the equation can be combined, combine them and use solution set notation. In this example, as -3 and $i\sqrt{19}$ are not like terms, we'll have to settle for the terms being uncombined.

Problems of the form $ax^2 + bx + c = 0$ with $a \neq 1$ require an initial step of dividing by a.

Domain & Range

When trying to grasp the intricacies of the interaction between two sets of numbers (we'll call them x and $f(x)$) in a relation, we call the x values that exist the **domain** of the relation; we call the $f(x)$ values that exist the **range** of the relation.

For example, Kate owns a store has a website that allows customers to purchase x shirts for y dollars:

Kate has to make decisions about how x is going to be restricted:

- Should Kate be interested orders for negative amounts of shirts? Probably not; that would imply Kate is buying shirts from the customer.
- Should Kate be interested in orders for zero shirts? I don't think so; would imply that the customer doesn't want to order anything.
- Should Kate be interested in orders for positive amounts of shirts? That sounds like business that Kate wants to be involved in!
- Should Kate be interested in orders for billions of shirts? How about trillions of shirts? How amount a number of shirts that would require Kate to use more matter than the planet has? Kate can only make so many shirts and only so quickly; there should probably be a maximum order size, restricting how many shirts can be ordered. Let's suppose that Kate only wants to commit to selling 500 shirts.
- Should Kate be interested in orders for portions of shirts? Orders for 2.8 shirts? Orders for π shirts? Only Kate would know if she wants to cut up portions of shirts and stuff them into boxes to ship to her customers. Let's assume she only wants to sell whole shirts.

All of the above choices limit the how many shirts Kate can sell on her website. All of the numbers of shirts that she's willing to sell, x, are considered the **domain** of her sales function. Based on the choices above, we could describe her domain as:

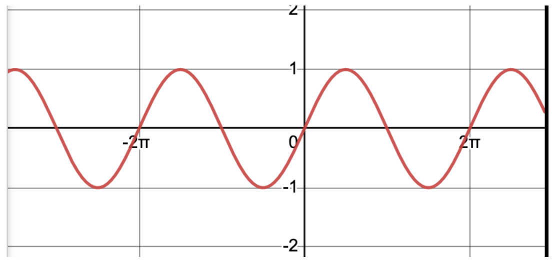
$D_f = \{1,2,3,\ldots,500\}$ or $D_f =$ whole numbers on $[1,500]$

If Kate sold her shirts for $8 apiece, then all of the amounts of money should could receive, which depends on how many shirts are ordered, are considered the **range** of her sales function. Based on the domain, we could describe her range as:

$R_f = \{8,16,24,\ldots,4000\}$ or $R_f =$ multiples of 8 on $[8,4000]$

When we graph a function, the domain is represented as values on the x-axis while the range is represented as values on the y-axis. Being able to understand what's happening to a function based on the graph, the story, or an algebraic expression is an essential skill!

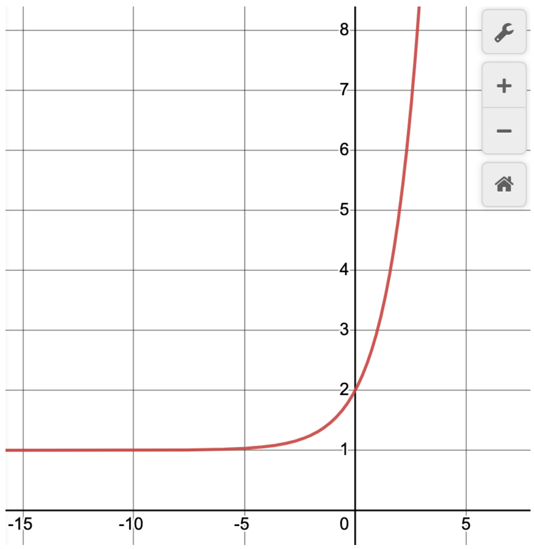
For each function or graph state the domain and range.

1) 

Domain: $(-\infty, \infty)$
Range: $[-1, 1]$

2) $y = x^2 + 3x + 2$

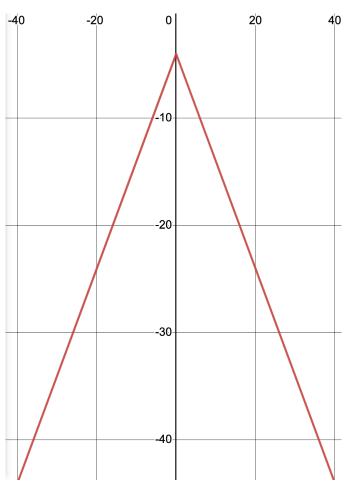
Domain: $(-\infty, \infty)$
Range: $[-.25, \infty)$

3) 

Domain: $(-\infty, \infty)$
Range: $(-1, \infty)$

4) $y = 3x + 4$

Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

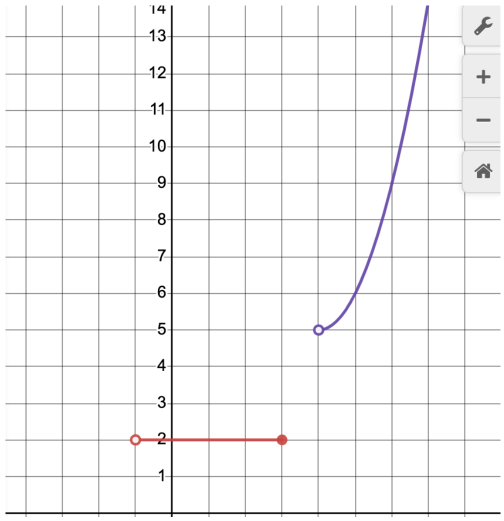
5) 

Domain: $(-\infty, \infty)$
Range: $(-\infty, -5]$

6) $y = |x - 4| + 3$

Domain: $(-\infty, \infty)$
Range: $[3, \infty)$

7)



Domain: $(-1,3)\ U\ (4,\infty)$

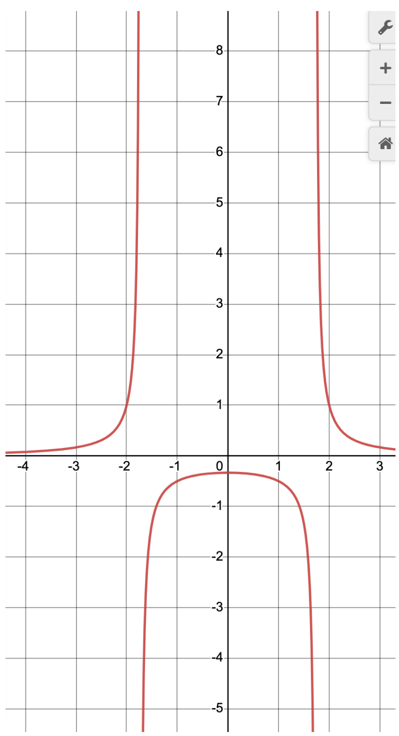
Range: $[2,2]\ U\ (5,\infty)$

8) $y = -2(x-1)^2 + 3$

Domain: $(-\infty,\infty)$

Range: $(-\infty,3]$

9)



Domain: $(-\infty,-2)\ U\ (-2,2)\ U\ (2,\infty)$

Range: $(-\infty,-.25) U (0,\infty)$

10) $y = \begin{cases} x+1, & x < -1 \\ 2, & x \geq 4 \end{cases}$

Domain: $(-\infty,-1) U [4,\infty)$

Range: $(-\infty, 0)$ U $[4,4]$